Section 2.4

Grade: «grade»

Subject: «subject»

Date: «date»

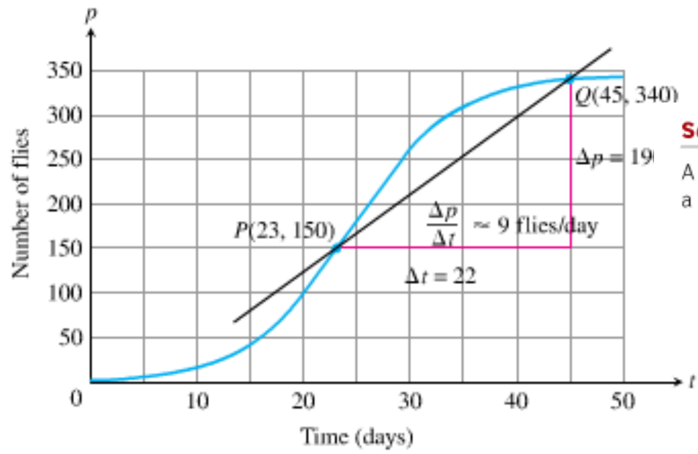
Lab: Zooming In

EXAMPLE 1 Finding Average Rate of Change

Find the average rate of change of $f(x) = x^3 - x$ over the interval [1, 3].

Slope of the secant

$$\frac{f(1) - f(3)}{1-3} = \frac{0-24}{1-3} = \frac{-24}{-2} = 12$$



Secant to a Curve

A line through two points on a curve is a **secant to the curve.**

Figure 2.27 Growth of a fruit fly population in a controlled experiment.
Source: Elements of Mathematical Biology. (Example 2)

EXAMPLE 2 Growing Drosophila in a Laboratory

Use the points $P(23, 150)$ and $Q(45, 340)$ in Figure 2.27 to compute the average rate of change and the slope of the secant line PQ.

$$\frac{340-150}{45-23} \approx 8.637 \text{ flies/day}$$

As suggested by Example 2, *we can always think of an average rate of change as the slope of a secant line.*

Secant-tangent of curve.gsp

Q	slope of line PQ

Tangent to a Curve

1. We start with what we can calculate, namely, the slope of a secant through P and a point Q nearby on the curve.

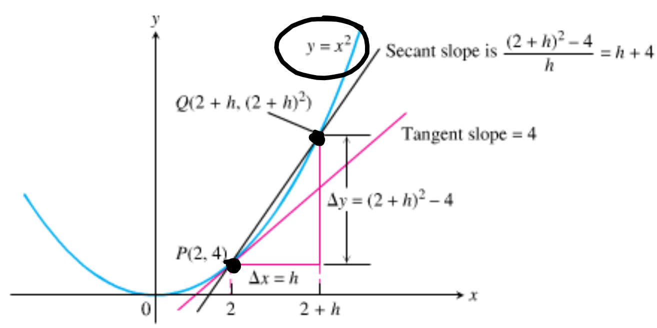
2. We find the limiting value of the secant slope (if it exists) as Q approaches P along the curve.

3. We define the *slope of the curve* at P to be this number and define the *tangent to the curve* at P to be the line through P with this slope.

EXAMPLE 3 Finding Slope and Tangent Line

Find the slope of the parabola $y = x^2$ at the point $P(2, 4)$. Write an equation for the tangent to the parabola at this point.

$y = x^2$



$$\lim_{h \to 0} \frac{(2+h)^2 - 4}{h}$$

$$\lim_{h \to 0} \frac{\cancel{2^2} + 4h + h^2 \cancel{- 4}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(4 + h)}{\cancel{h}}$$

$$\lim_{h \to 0} 4 + h = 4$$

DEFINITION **Slope of a Curve at a Point**

The **slope of the curve** $y = f(x)$ at the point $P(a, f(a))$ is the number

$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h},$$

provided the limit exists.

EXAMPLE 4 Exploring Slope and Tangent

Let $f(x) = 1/x$.

(a) Find the slope of the curve at $x = a$.

(b) Where does the slope equal $-1/4$?

(c) What happens to the tangent to the curve at the point $(a, 1/a)$ for different values of a?

$-\frac{1}{a^2} = -\frac{1}{4}$ when a or $x = 2$

$$\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$\lim_{h\to 0} \frac{\frac{1}{a+h} - \frac{1}{a}}{h}$$

The expression

$$\frac{f(a+h) - f(a)}{h}$$

is the **difference quotient of f at a.**

$$\lim_{h\to 0} \frac{\frac{a-(a+h)}{a(a+h)}}{h} = \lim_{h\to 0} \frac{-\cancel{h}}{a(a+h)} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{-1}{a(a+h)}$$

$$= \frac{-1}{a^2}$$

1 Find the slope at the point P(2,4) of

$$y = x^2$$

Normal to a Curve

The **normal line** to a curve at a point is the line perpendicular to the tangent at that point.

EXAMPLE 5 Finding a Normal Line

Write an equation for the normal to the curve $f(x) = 4 - x^2$ at $x = 1$.

2 Answer?

$$\lim_{h \to 0} \frac{f(t+h) - f(t)}{h}.$$

EXAMPLE 6 Investigating Free Fall

Find the speed of the falling rock in Example 1, Section 2.1, at $t = 1$ sec.